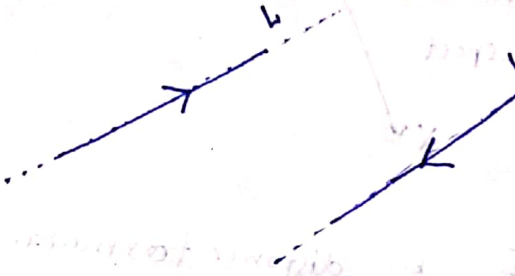
Unit 4

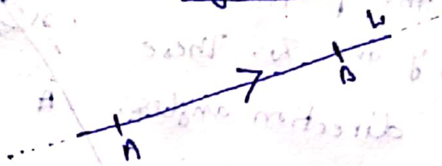
Vectors.

* Basic Concepts

Let 'L' be any straight line in plane. This line can be given two directions by arrowheads.

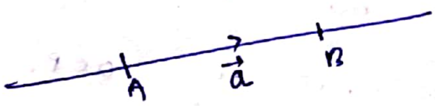


A line with one of these direction is called a directed line.
Now, if we restrict the line L to the line segment AB, then a magnitude (length of the segment AB) with direction on the line L.
So, we have a directed line segment.



* Definition of vector.

A quantity that has magnitude as well as direction is called a vector. Here line segment AB is a vector, denoted by $\overrightarrow{AB}$ or a vector $\vec{a}$



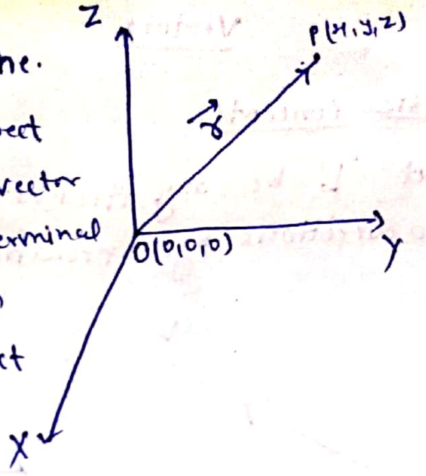
point A is start called initial point of a vector
point B is end called terminal point of a vector

* magnitude.

The distance between initial and terminal points of a vector is called the magnitude or length of the vector, denoted as $|\overrightarrow{AB}|$ or $|\vec{a}|$.

Note:- Since the length is never negative, the notation $|\vec{a}| < 0$ has no meaning.

* position vector

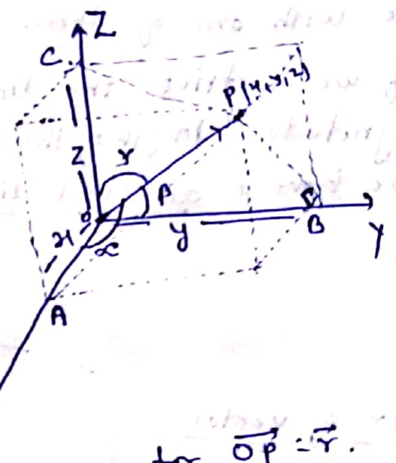


Consider a point P in XYZ (space) plane having co-ordinates (x,y,z) with respect to the origin O(0,0,0). Then $\vec{OP}$ is a vector having O as initial point and P as terminal point. Vector $\vec{OP}$ is called position vector of the point P with respect to O.

the magnitude of $\vec{OP}$ or $\vec{r}$ is

$$|\vec{OP}| = |\vec{r}| = \sqrt{x^2+y^2+z^2}$$ by distance formula.

* Direction cosine.



Consider the position vector $\vec{OP}$ or $\vec{r}$, vector $\vec{r} = \vec{OP}$ makes angle α, β and γ with the positive direction of x, y and z. These angles are called direction angles.

The cosine of these angles as $\cos\alpha$, $\cos\beta$ and $\cos\gamma$ are called direction cosine of position vector $\vec{OP} = \vec{r}$.

Now from Right angle traingle OAP, $\cos\alpha = \frac{x}{r}$ $\quad$ ($\because r = |\vec{r}|$)

" " " " OBP, $\cos\beta = \frac{y}{r}$ $\quad$ —— (*)

" " " " OCP, $\cos\gamma = \frac{z}{r}$

From (*), $\quad x = r\cos\alpha = lr$ $\quad$ [where $l = \cos\alpha$
$\quad y = r\cos\beta = mr$ $\quad$ $m = \cos\beta$
$\quad z = r\cos\gamma = nr$ $\quad$ $n = \cos\gamma$] —— (**)

Thus co-ordinates of point P may be written as $(x,y,z) = (lr, mr, nr)$ proportional to the direction cosines are

Here the numbers lr, mr and nr called direction ratios denoted by a, b and c.

i.e $\{a = lr,\ b = mr,\ c = nr.\}$ —— (***)
$\quad \downarrow x \quad \downarrow y \quad \downarrow z.$ (by *)

Example: write the direction ratios of the vector $\vec{a} = \vec{i} + \vec{j} - 2\vec{k}$. and calculate direction cosine.

solution:- The direction ratios a, b, c of $\vec{r} = x\vec{i} + y\vec{j} + z\vec{k}$ and by question, $x = 1$, $y = 1$, $z = -2$ Hence. $a = 1$, $b = 1$, $c = -2$. (by **)

Now, direction cosine by (***)

$l = \frac{a}{r} = \frac{a}{|\vec{r}|} = \frac{1}{\sqrt{1^2+1^2+(-2)^2}} = \frac{1}{\sqrt{6}}$

lly $\quad m = \frac{b}{r} = \frac{1}{\sqrt{6}}$

$n = \frac{c}{r} = -2/\sqrt{6}$

$\therefore (l, m, n) = \left(\frac{1}{\sqrt{6}}, \frac{1}{\sqrt{6}}, -\frac{2}{\sqrt{6}}\right)$

* Types of vectors

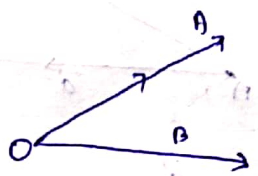
(i) **Zero vector** :- A vector whose initial and terminal points coincide. It is denoted by $\vec{0}$. The vectors $\vec{AA}$, $\vec{BB}$ are zero vectors.

(ii) **Unit vector** :- A vector whose magnitude is unity (1).
The unit vector in the direction of a vector $\vec{a}$ is denoted by $\hat{a}$ (a-cap or a-hat).
if $\vec{a} = x\vec{i} + y\vec{j} + z\vec{k}$ then

$$\hat{a} = \frac{\vec{a}}{|\vec{a}|} = \frac{x}{\sqrt{x^2+y^2+z^2}}\hat{i} + \frac{y}{\sqrt{x^2+y^2+z^2}}\hat{j} + \frac{z}{\sqrt{x^2+y^2+z^2}}\hat{k}.$$

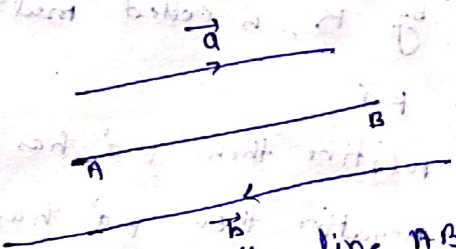
(iii) **Co-initial vectors** :- Two or more vectors having the same initial point.

Ex:



Here $\vec{OA}$ and $\vec{OB}$ are co-initial vectors.

(iv) **Collinear vectors** :-
Two or more vectors are said to be co-linear if they are parallel to the same line, irrespective of their magnitude and direction



vector $\vec{a}$ and $\vec{b}$ are parallel to the line AB, so they are collinear.

(v) **Equal vector** :-
Two vector $\vec{a}$ and $\vec{b}$ are equal if they have same magnitude and direction, written as $\vec{a} = \vec{b}$.

(vi) **Negative of a vector**:
A vector whose magnitude is same but direction is opposite.
as $\vec{BA} = -\vec{AB}$.

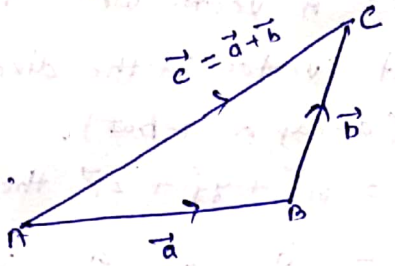
Addition of vectors

A vector $\vec{AB}$ means the displacement from point A to point B. Suppose an object moves from A to B and then from B to C. The net displacement made by object from point A to point C given by $\vec{AC}$.

$$\vec{AC} = \vec{AB} + \vec{BC}.$$

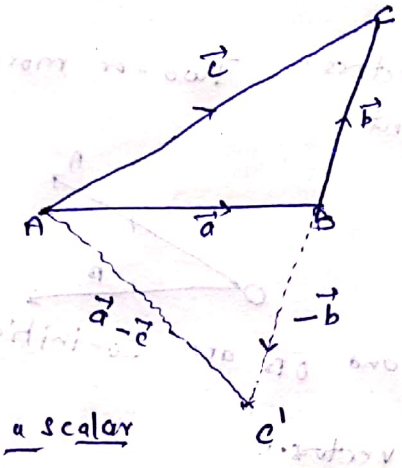
or $\vec{c} = \vec{a} + \vec{b}$

This is known as triangle law of vector addition.



* Subtraction of vectors

$\vec{AC} = \vec{AB} - \vec{BC'}$
$= \vec{a} - \vec{c}.$

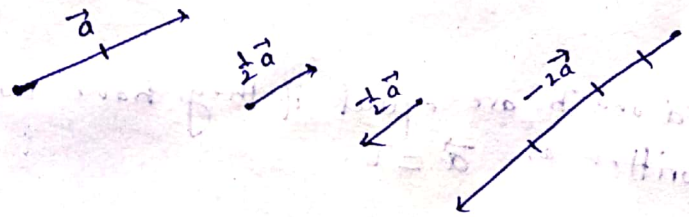


* multiplication of vector by a scalar

let $\vec{a}$ be the vector and K be the scalar, then the product of vector $\vec{a}$ by K, is called multiplication of vector by scalar denoted by $K\vec{a}$.

if K is positive then $K\vec{a}$ has same direction as $\vec{a}$.

if K is negative then $K\vec{a}$ has oposite direction as $\vec{a}$.



note: $|K\vec{a}| = |K||\vec{a}|$.