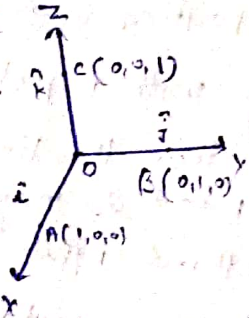


* Component of a vector



Take the points A(1,0,0), B(0,1,0) and C(0,0,1) on the x-axis, y-axis and z-axis.
then $|\vec{OA}| = 1$, $|\vec{OB}| = 1$ and $|\vec{OC}| = 1$.
That each has magnitude 1.
These are called unit vectors along the axes OX, OY and OZ, and denoted by $\hat{i}$, $\hat{j}$ and $\hat{k}$.

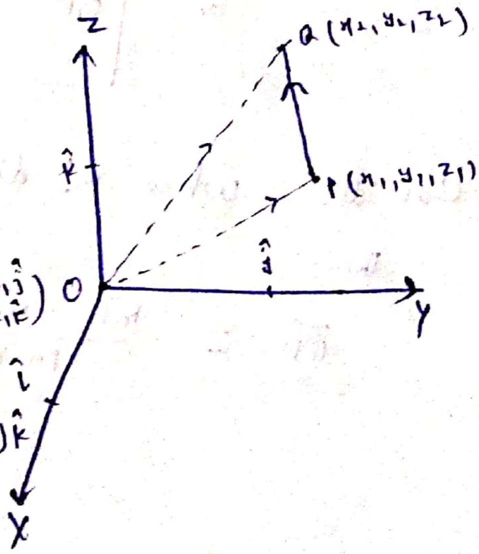
Thus position vector of a point P(x, y, z) in XOY plane with refrence to O is given by

$$\vec{OP} = \vec{r} = x\hat{i} + y\hat{j} + z\hat{k}.$$

and $$|\vec{OP}| = |\vec{r}| = \sqrt{x^2+y^2+z^2}.$$

* Vector joining two points

if $P(x_1, y_1, z_1)$ and $Q(x_2, y_2, z_2)$ are any two points, then vector joining P and Q is the vector $\vec{PQ}$



By triangle law,

$$\vec{OP} + \vec{PQ} = \vec{OQ}$$

or $$\vec{PQ} = \vec{OQ} - \vec{OP}$$

$$= (x_2\hat{i} + y_2\hat{j} + z_2\hat{k}) - (x_1\hat{i} + y_1\hat{j} + z_1\hat{k})$$

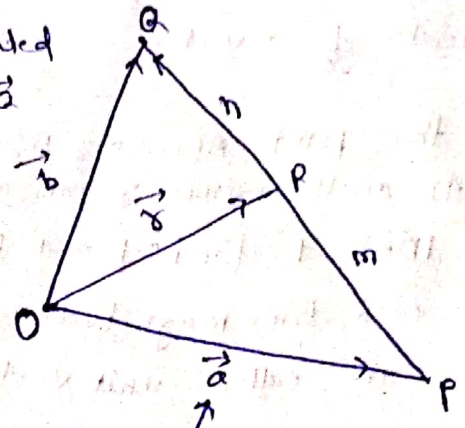
$$= (x_2-x_1)\hat{i} + (y_2-y_1)\hat{j} + (z_2-z_1)\hat{k}$$

and magnitude of $\vec{PQ}$ is

$$|\vec{PQ}| = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2 + (z_2-z_1)^2}$$

* Section formula

Let P and Q be two points represented by the position vector $\vec{OP}$ and $\vec{OQ}$ respectively with respect to the origin O. Then the line segment joining P and Q may divided by a third point R as internally or externally.



Case I. when R divides PQ internally (as above figure)

then $m\vec{RQ} = n\vec{PR}$ (i) $\left[\because \frac{RQ}{RP} = \frac{n}{m}\right]$

since $\vec{RQ} = \vec{OQ} - \vec{OR} = \vec{b} - \vec{r}$

and $\vec{PR} = \vec{OR} - \vec{OP} = \vec{r} - \vec{a}$

therefore from (i)

$$m(\vec{b}-\vec{r}) = n(\vec{r}-\vec{a})$$

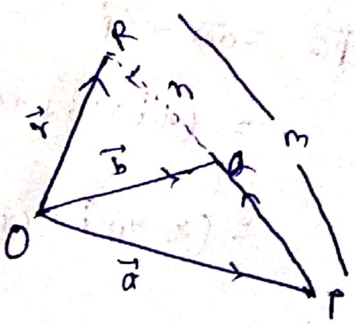
or $$\vec{r} = \frac{m\vec{b}+n\vec{a}}{m+n}$$

or $$\boxed{\vec{OR} = \frac{m\vec{b}+n\vec{a}}{m+n}}$$

Case-II. when R divides PQ externally.

then

$$\vec{OR} = \frac{m\vec{b}-n\vec{a}}{m-n}$$



Note:- when $m=n$ then $\vec{OR} = \frac{\vec{a}+\vec{b}}{2}$

Some examples :-

(1.) Compute the magnitude of vector $\vec{a} = \hat{i} + \hat{j} + \hat{k}$.

Solution:. $|\vec{a}| = \sqrt{1^2+1^2+1^2} = \sqrt{3}$

(2). Find the values of x and y so that the vectors $2\hat{i} + 3\hat{j}$ and $x\hat{i} + y\hat{j}$ are equal.

Solution:

$$2\hat{i} + 3\hat{j} = x\hat{i} + y\hat{j} \Rightarrow x = 2, \; y = 3.$$

(3) Find the sum of the vectors $\vec{a} = \hat{i} - 2\hat{j} + \hat{k}$; $\vec{b} = -2\hat{i} + 4\hat{j} + 5\hat{k}$ and $c = \hat{i} - 6\hat{j} - 7\hat{k}$

Solution:.

$$\vec{a} + \vec{b} + \vec{c} = (1-2+1)\hat{i} + (-2+4-6)\hat{j} + (1+5-7)\hat{k}$$
$$= 0\hat{i} - 4\hat{j} - 1\hat{k}$$
$$= -4\hat{j} - \hat{k}$$

(4) Find the unit vector in the direction of the vector $\vec{a} = \hat{i} + \hat{j} + 2\hat{k}$

Solution:.

$$|\vec{a}| = \sqrt{1^2+1^2+2^2} = \sqrt{6}$$
$$\therefore \quad \hat{a} = \frac{\vec{a}}{|\vec{a}|} = \frac{\hat{i} + \hat{j} + 2\hat{k}}{\sqrt{6}} = \frac{1}{\sqrt{6}}\hat{i} + \frac{1}{\sqrt{6}}\hat{j} + \frac{2}{\sqrt{6}}\hat{k}$$

(5) Find the unit vector in the direction of vector $\vec{PQ}$, where P and Q are the points (1,2,3) and (4,5,6) respectively.

Solution:.

$$\vec{PQ} = (4-1)\hat{i} + (5-2)\hat{j} + (6-3)\hat{k} = 3\hat{i} + 3\hat{j} + 3\hat{k}$$
$$|\vec{PQ}| = \sqrt{3^2+3^2+3^2} = \sqrt{27} = 3\sqrt{3}$$

So, unit vector $= \dfrac{\vec{PQ}}{|\vec{PQ}|} = \dfrac{3\hat{i} + 3\hat{j} + 3\hat{k}}{3\sqrt{3}} = \dfrac{1}{\sqrt{3}}\hat{i} + \dfrac{1}{\sqrt{3}}\hat{j} + \dfrac{1}{\sqrt{3}}\hat{k}$

(6) For vectors $\vec{a} = 2\hat{i} - \hat{j} + 2\hat{k}$ and $\vec{b} = -\hat{i} + \hat{j} - \hat{k}$, find the unit vector in the direction of the vector $\vec{a} + \vec{b}$.

Solution:.

$$\vec{a} + \vec{b} = (2-1)\hat{i} + (-1+1)\hat{j} + (2-1)\hat{k} = \hat{i} + \hat{k}$$

So, unit vector $= \dfrac{(\vec{a} + \vec{b})}{|\vec{a} + \vec{b}|} = \dfrac{\hat{i} + \hat{k}}{\sqrt{2}} = \dfrac{1}{\sqrt{2}}\hat{i} + \dfrac{1}{\sqrt{2}}\hat{k}$.

(7) Find a vector in the direction of vector $5i - j + 2k$ which has magnitude 8 units.

Solution:. let $\vec{a} = 5\hat{i} - \hat{j} + 2k$

$$|\vec{a}| = \sqrt{5^2 + (-1)^2 + 2^2} = \sqrt{25+1+4} = \sqrt{30}$$

$$\therefore \hat{a} = \frac{\vec{a}}{|\vec{a}|} = \frac{5i - j + 2k}{\sqrt{32}}$$

So, a vector parallel to $5i - j + 2k$ with magnitude 8 units is

$$8\hat{a} = 8\left(\frac{5i - j + 2K}{\sqrt{30}}\right) = \frac{40}{\sqrt{3}}i - \frac{8}{\sqrt{3}}j + \frac{16}{\sqrt{30}}K$$

(8) Show that the vectors $2i - 3j + 4K$ and $-4i + 6j - 8K$ are collinear.

Solution:. let $\vec{a} = 2i - 3j + 4K$, $\vec{b} = -4i + 6j - 8K$

Here $\vec{b} = -4i + 6j - 8K = -2(2i - 3j + 4K) = -2\vec{a}$

$\therefore \vec{b} = \lambda\vec{a}$. for $\lambda = -2$

So, the vectors are collinear.

(9) Find the direction cosine of vector $i + 2j + 3K$

Solution:. let $\vec{a} = i + 2j + 3K$ and

$$|\vec{a}| = \sqrt{1^2 + 2^2 + 3^2} = \sqrt{14}$$

So the DC' of $\vec{a}$ are $\left(\frac{1}{\sqrt{14}}, \frac{2}{\sqrt{14}}, \frac{3}{\sqrt{14}}\right)$

(10) Find the direction cosines of the vector joining the point $P(1,2,-3)$ and $Q(-1,-1,1)$ directed from P to Q.

Solution:.

$$\vec{PQ} = (-1-1)i + (-2-2)j + (1-(-3))K$$
$$= -2i - 4j + 4K$$

$$|\vec{PQ}| = \sqrt{(-2)^2 + (-4)^2 + 4^2} = \sqrt{36} = 6$$

So DC' of $\vec{AB}$ are $\left(-\frac{2}{6}, -\frac{4}{6}, \frac{4}{6}\right) = \left(-\frac{1}{3}, \frac{-2}{3}, \frac{2}{3}\right)$.

(11). Show that the vector $i+j+k$ is equally inclined to axes OX, OY and OZ.

Solution:. let $\vec{a} = i+j+k$ then

$$|\vec{a}| = \sqrt{1^2+1^2+1^2} = \sqrt{3}$$

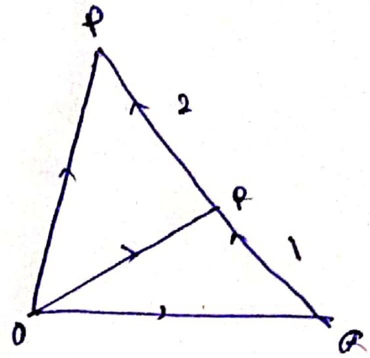
So the Direction cosine of $\vec{a}$ are $\left(\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right)$

i.e $\cos\alpha = \frac{1}{\sqrt{3}}$, $\cos\beta = \frac{1}{\sqrt{3}}$ and $\cos\gamma = \frac{1}{\sqrt{3}}$

So the vector is equally inclined to OX, OY and OZ.

(12) Find the position vector of a point R which divides the line joining two point P and Q whose position vectors are $i+2j-k$ and $-i+j+k$ respectively in the ratio 2:1, internally and externally.

Solution:. let



$$\vec{OP} = i+2j-k \text{ and}$$
$$\vec{OQ} = -i+j+k$$

(i) Internally.

position vector of R is

$$\vec{OR} = \frac{2(-i+j+k)+1(i+2j-k)}{2+1} = -\frac{1}{3}i+\frac{4}{3}j+\frac{1}{3}k$$

(ii) Externally

$$\vec{OR} = \frac{2(-i+j+k)-1(i+2j-k)}{2-1} = -3i+3k.$$