Name - Dr. V. V. Rajaly
Date : 03-04-2020
Period : 1st

Vector (Unit-04) Lec-05

Sem - Civil + EC
Subject - Basic math
Code - 1601103 / 1602203

* APPLICATIONS OF VECTORS :—

* **WORK DONE :-** WORK done by a force on a body is defined as the product of the component of the force in the direction in the displacement and displacement.

WORK DONE :— (Components of F along displacement) × (displacement)

$$\therefore \quad W = F\cos\theta \cdot AB$$

$$= \vec{F} \cdot \vec{AB}$$

$$= \vec{F} \cdot \vec{d}$$

Work done = force . displacement

Note : when more than two forces acting at a body we find their resultant.

by $\vec{F_R} = \vec{F_1} + \vec{F_2} + \ldots$

* MOMENT OF FORCE ABOUT A POINT :-

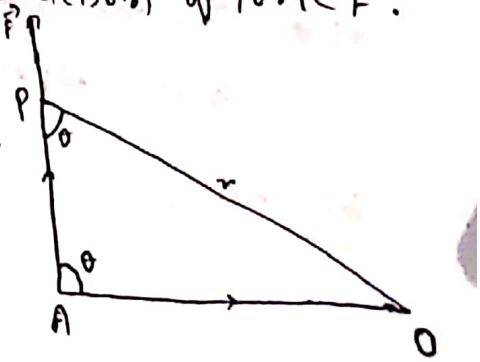
Consider any point, P on the line of action of force $\vec{F}$.

Let O be any point with $\vec{OP} = \vec{r}$

Draw OA perpendicular to the line of action $\vec{F}$.

Then moment of force is defined as Moment $= |\vec{F}||\vec{OA}|$

$= |F| r \sin\theta$



Moment of force $= \vec{r} \times \vec{F}$

Moment of force acting at P about a point O is

$= \vec{r} \times \vec{F} = \vec{OP} \times \vec{F}$

* MOMENT OF A FORCE ABOUT A LINE :—

Moment of force $\vec{F}$ about a line is resolved part along the line of the moment of $\vec{F}$ about any point on the line.

$$\vec{a} = a_1\vec{i} + a_2\vec{j} + a_3\vec{k}$$

Then unit vector parallel to the line

$$\hat{a} = \frac{a_1\hat{i} + a_2\hat{j} + a_3\hat{k}}{\sqrt{a_1^2 + a_2^2 + a_3^2}}$$

Moment of force about the line m.

$$\hat{e} = m \cdot \frac{a_1\hat{i} + a_2\hat{j} + a_3\hat{k}}{\sqrt{a_1^2 + a_2^2 + a_3^2}}$$

EXAMPLE

Q1.→ Find the work done by the force $\vec{F} = 3\vec{i} + 2\vec{j} - 4\vec{k}$ given that displacement produced by the force is $2\vec{i} + 3\vec{j} + \vec{k}$.

Solution:- Force $\vec{F} = 3\vec{i} + 2\vec{j} - 4\vec{k}$ and displacement $d = 2\vec{i} + 3\vec{j} + \vec{k}$

work done = force . displacement

$W = \vec{F} \cdot \vec{d}$

$= (3\vec{i} + 2\vec{j} - 4\vec{k}) \cdot (2\vec{i} + 3\vec{j} + \vec{k}) = 8$ units. Ans.

Ex!. Two forces $\hat{i}+\hat{j}-\hat{k}$ and $-2\hat{i}-\hat{j}+\hat{k}$ are applied at the point $3\hat{i}-\hat{j}$. Find the moment of the force system about the point $2\hat{i}+\hat{j}-2\hat{k}$.

solution!. Let $\vec{F_1} = \hat{i}+\hat{j}-\hat{k}$

$\vec{F_2} = -2\hat{i}-\hat{j}+\hat{k}$

Resultant force, $\vec{F} = \vec{F_1} + \vec{F_2}$

$= (\hat{i}+\hat{j}-\hat{k}) + (-2\hat{i}-\hat{j}+\hat{k})$

$= -\hat{i}+0\hat{j}+0\hat{k}$

Now $\vec{r} = (3\hat{i}-\hat{j}+0\hat{k}) - (2\hat{i}+\hat{j}-2\hat{k})$

$\vec{r} = \hat{i}-2\hat{j}+2\hat{k}$

moment of force $\vec{M} = \vec{r} \times \vec{F}$

$= (\hat{i}-2\hat{j}+2\hat{k}) \times (-\hat{i}+0\hat{j}+0\hat{k})$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -2 & 2 \\ -1 & 0 & 0 \end{vmatrix} = \hat{i}(0-0)-\hat{j}(0+2)+\hat{k}(0-2)$$

$= 0\hat{i}-2\hat{j}-2\hat{k}$

magnitude of moment

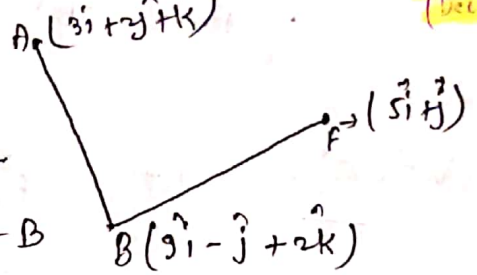
$M = |0\hat{i}-2\hat{j}-2\hat{k}| = \sqrt{(-2)^2+(-2)^2}$

$= \sqrt{4+4}$

$= \sqrt{8}$

$= 2\sqrt{2}$ units. Ans.

Ex!. The force represented by $5\hat{i}+\hat{j}$ is acting through the point $9\hat{i}-\hat{j}+2\hat{k}$. Find the moment about the point $3\hat{i}+2\hat{j}+\hat{k}$. (Dec-2004)



Solution: Let A be a given point about which the moment is to be obtained and let B be the point on the line of action of force. Then the position vector of $B = 9\hat{i}-\hat{j}+2\hat{k}$ and the position vector of $A = 3\hat{i}+2\hat{j}+\hat{k}$.

$$\therefore \vec{r} = \overrightarrow{AB}$$

$$= (9\hat{i}-\hat{j}+2\hat{k}) - (3\hat{i}+2\hat{j}+\hat{k})$$

$$= (6\hat{i}-3\hat{j}+\hat{k}) \quad \text{---(1)}$$

$$\vec{F} = 5\hat{i}+\hat{j} \quad \text{---(11)}$$

$$\therefore M = \vec{r} \times \vec{F} = (6\hat{i}-3\hat{j}+\hat{k})(5\hat{i}+\hat{j})$$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 6 & -3 & 1 \\ 5 & 1 & 0 \end{vmatrix}$$

$$= \hat{i}(-3\times 0 - 1) - \hat{j}(0-5) + \hat{k}(6+15)$$

$$= -\hat{i} + 5\hat{j} + 21\hat{k}$$

Magnitude of moment $= |\vec{M}| = \sqrt{1+25+441}$

$$= \sqrt{467} \text{ units}$$ Ans.

Ex!: Find the moment about the point M(-2, 4, -6) of the force represented in magnitude and position by $\vec{AB}$ where the point A and B have the co-ordinates (1, 2, -3) and (3, -4, 2) respectively. (Sikkim Dec 2006)

Solution!: $\vec{F} = \vec{AB}$

$= (3\hat{i} - 4\hat{j} + 2\hat{k}) - (\hat{i} + 2\hat{j} - 3\hat{k})$

$= 2\hat{i} - 6\hat{j} + 5\hat{k}$

$\vec{r} = \vec{MA} = (\hat{i} + 2\hat{j} - 3\hat{k}) - (-2\hat{i} + 4\hat{j} - 6\hat{k})$

$= 3\hat{i} - 2\hat{j} + 3\hat{k}$

moment $\vec{M} = \vec{r} \times \vec{F}$

$= (3\hat{i} - 2\hat{j} + 3\hat{k}) \times (2\hat{i} - 6\hat{j} + 5\hat{k})$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & -2 & 3 \\ 2 & -6 & 5 \end{vmatrix}$$

$= \hat{i}(-10 + 18) - \hat{j}(15 - 6) + \hat{k}(-18 + 4)$

$= 8\hat{i} - 9\hat{j} - 14\hat{k}$

magnitude of the moment $= \sqrt{(8)^2 + (-9)^2 + (-14)^2}$

$= \sqrt{64 + 81 + 196}$

$= \sqrt{341}$

D.C.'s $= \frac{8}{\sqrt{341}}, \frac{-9}{\sqrt{341}}, \frac{-14}{\sqrt{341}}$ Ans.

Exercise

Q.1. Find the work done by the force $\vec{F} = 3\hat{i} + 2\hat{j} - 4\hat{k}$ given that displacement produced by the force is $\hat{i} + 2\hat{j} - 3\hat{k}$.

Ans - 17 Units.

Q.2. Find the moment about the line A(3, -1, 3) of a force $(2\hat{i} + 2\hat{j} + \hat{k})$ acting at B(5, 2, 4).

Ans: - $\sqrt{101}$ Units.

Q.3. A force $\vec{F} = 4\hat{i} + 4\hat{k}$ acts through a point A(0, 2, 0). Find the moment of F about a point B(4, 0, 4).

(Dec 2004)

Q.4. Forces $2\hat{i} + 7\hat{j}$, $2\hat{i} - 5\hat{j} + 6\hat{k}$, $-\hat{i} + 2\hat{j} - \hat{k}$ act on a point P whose position vector is $4\hat{i} - 3\hat{j} - 2\hat{k}$. Find the vector moment of the resultant of three forces acting at P about the point B, whose vector is $6\hat{i} + \hat{j} - 3\hat{k}$.

Ans: $-24\hat{i} + 13\hat{j} + 7\hat{k}$

(Diploma Dec 2005)

Q.5. Find the moment about the point $2\hat{i} - \hat{j} + 3\hat{k}$ of a force represented by $\hat{i} - \hat{j} + 2\hat{k}$ through the point $3\hat{i} + \hat{j} - \hat{k}$.

(Diploma 2010)